Average, Marginal Business and Economics Functions

Marginal Cost – The rate of change of the cost of producing x items: C'(x)

Average Cost – The average cost of each item among x items: $\bar{C}(x) = \frac{C(x)}{x}$

Marginal Average Cost – The rate of change of the average cost of each item among x items:

$\bar{C}'(x)$

Example: Bill owns a small business for selling pizza. The cost of producing x pizzas is given by the function $C(x) = .5x^2 - 4x + 8$

a) Find the approximate cost of producing the 11th pizza:

i) Find C'(x) = $\mathbf{0.1x - 4}$

ii) Find C'(10) = $10 - 4 =$ **$6** = The approximate cost of producing the 11th pizza.

b) Find the average cost per pizza for producing 20 pizzas:

i) Find $\bar{C}(x) = \frac{C(x)}{x} = \frac{0.5x^2}{x} - \frac{4x}{x} + \frac{8}{x} = \mathbf{0.5x - 4 + \frac{8}{x}}$

ii) Find $\bar{C}(20) = 0.5(20) - 4 + \frac{8}{20} = 10 - 4 + 0.4 = \$\mathbf{6.40}$ = The average cost per pizza for producing 20 pizzas.

c) Find the average cost of producing the 12th pizza:

i) Find $\bar{C}'(x) = 0.5 - \frac{8}{x^2}$

ii) Find $\bar{C}'(11) = 0.5 - \frac{8}{11^2} = 0.5 - \frac{8}{121} = 0.5 - .066 = \$0.43 =$ The average cost per pizza for producing the 12 pizza.

Marginal Revenue – The rate of change for the revenue for producing x items: $R'(x)$

Average Revenue – The average revenue of each item from among x items: $\bar{R}(x) = \frac{R(x)}{x}$

Marginal Average Revenue – The rate of change of the average revenue of each item among x items:

$\bar{R}'(x)$

Example: Bill owns a small business for selling pizza. The revenue for producing x pizzas is given by the function $R(x) = -.2x^2 + 2x$

a) Find the approximate revenue for producing the 8th pizza:

i) Find R'(x) = $\mathbf{-0.4x + 2}$

ii) Find R'(7) = $-0.4(7) + 2 = -\$\mathbf{0.80}$ = The approximate revenue for producing the 11th pizza.

b) Find the average revenue per pizza for producing 8 pizzas:

i) Find $\bar{R}(x) = \frac{R(x)}{x} = \frac{-0.2x^2}{x} + \frac{2x}{x} = \mathbf{-0.2x + 2}$

ii) Find $\bar{R}(8) = -0.2(8) + 2 = -1.60 + 2 = \$\mathbf{0.40}$ = The average revenue per pizza for producing 8 pizzas.

c) Find the average revenue for producing the 8th pizza:

i) Find $\bar{R}'(x) = \mathbf{-0.2}$

ii) Find $\bar{R}'(7) = -\$\mathbf{0.20}$ = The average revenue per pizza for producing the 8th pizza.

Marginal Profit – The rate of change for the profit for producing x items: $P'(x)$

Average Profit – The average profit of each item from among x items: $\bar{P}(x) = \frac{P(x)}{x}$

Marginal Average Profit – The rate of change of the average profit of each item among x items:

$\bar{P}'(x)$

Example: Bill owns a small business for selling pizza. The revenue for producing x pizzas is given by the function $P(x) = -.7x^2 + 6x - 8$

a) Find the approximate profit for producing the 4th pizza:

i) Find P'(x) = $\mathbf{-1.4x + 6}$

ii) Find P'(3) = $-1.4(3) + 6 = \$\mathbf{1.80}$ = The approximate profit for producing the 4th pizza.

b) Find the average profit per pizza for producing 4 pizzas:

i) Find $\bar{P}(x) = \frac{P(x)}{x} = \frac{-0.7x^2}{x} + \frac{6x}{x} - \frac{8}{x} = \mathbf{-0.7x + 6 - \frac{8}{x}}$

ii) Find $\bar{P}(4) = -0.7(4) + 6 - \frac{8}{4} = -2.8 + 6 - 2 = \$\mathbf{1.20}$ = The average profit per pizza for producing 4 pizzas.

c) Find the average profit per pizza for producing the 6th pizza:

i) Find $\bar{P}'(x) = -0.7 - \frac{8}{x^2}$

ii) Find $\bar{P}'(5) = -0.7 - \frac{8}{5^2} = -0.7 - \frac{8}{25} = -0.7 - 0.32 = -\$\mathbf{1.02}$ = The average profit per pizza for producing the 6th pizza.

Practice Problems:

John owns a small business selling High Definition Televisions. His Cost, Revenue and Profit functions are given by the following:

$$\boldsymbol{R(x) = -0.05x^2 + 250x} \qquad \boldsymbol{C(x) = 125{,}000 + 35x} \qquad \boldsymbol{P(x) = -0.05x^2 + 215x - 125{,}000}$$

a) Find the cost of producing the 1001th TV.

b) Find the average cost per TV for producing 1000 TVs.

c) Find the average cost per TV for producing the 1001th TV.

d) Find the revenue for producing the 1001th TV.

e) Find the average revenue per TV for producing 1000 TVs.

f) Find the average revenue per TV for producing the 1001th TV.

g) Find the profit for producing the 1001th TV.

h) Find the average profit per TV for producing 1000 TVs.

i) Find the average profit per TV for producing the 1001th TV.

Answer Key:

a) $35
b) $160
c) $.035
d) $150.10
e) $200
f) $0.15
g) $115.10
h) $40
i) $.115