Linear approximation of a function value at a given point.

Steps

1. Find the derivative and substitute the given x value to find $f'(a)$.
2. Substitute the given x value to the original equation to find $f(a)$.
3. Use the equation $y = f(a) + f'(a)(x - a)$.

Example 1

Use the equation $f(x) = \sqrt{x}$ at $x = 4$ to approximate $\sqrt{3.9}$.

Step 1) Find the derivative and substitute the given x value to find $f'(a)$.

$f(x) = \sqrt{x}$	Original equation
$f(x) = x^{1/2}$	Write in exponential form.
$f'(x) = \frac{1}{2}x^{-1/2}$	Use the power rule
$f'(x) = \frac{1}{2\sqrt{x}}$	Rewrite in radical form.
$f'(4) = \frac{1}{2\sqrt{4}}$	Substitute x value and solve
$f'(4) = \frac{1}{4}$	Find $f'(a)$

Step 2) Substitute the given x value to the original equation to find $f(a)$.

$f(x) = \sqrt{x}$	Original equation
$f(4) = \sqrt{4}$	Substitute x value and solve
$f(4) = 2$	Find $f(a)$

Step 3) Use the equation $y = f(a) + f'(a)(x - a)$ where $f(a)$ is the y value of the original equation (2), $f'(a)$ is the y value of the derivative ($\frac{1}{4}$), x is the value you want to approximate (3.9), and a is the value you are using to approximate (4).

$$y = f(a) + f'(a)(x - a)$$

$$y = 2 + \frac{1}{4}(3.9 - 4)$$

$$y = 1.975$$

Comparing this with the decimal approximation the calculator gives, it is pretty close.

$$\sqrt{3.9} \approx 1.9748417$$

Example 2

Use the equation $f(x) = \tan(x)$ where $x = \frac{\pi}{4}$ to approximate $\tan(0.8)$.

Step 1) Find the derivative and substitute the given x value to find $f'(a)$.

$f(x) = \tan(x)$	Original equation
$f'(x) = sec^2(x)$	Take derivative
$f'(x) = sec^2(\frac{\pi}{4})$	Substitute x value and solve
$f'(x) = 2$	Find $f'(a)$

Step 2) Substitute the given x value to the original equation to find $f(a)$.

$f(x) = \tan(x)$	Original equation
$f\left(\frac{\pi}{4}\right) = \tan(\frac{\pi}{4})$	Substitute x value and solve
$f\left(\frac{\pi}{4}\right) = 1$	Find $f(a)$

Step 3) Use the equation $y = f(a) + f'(a)(x - a)$ where $f(a)$ is the y value of the original equation (1), $f'(a)$ is the y value of the derivative (2), x is the value you want to approximate (0.8), and a is the value you are using to approximate ($\frac{\pi}{4}$).

$$y = f(a) + f'(a)(x - a)$$

$$y = 1 + 2(0.8 - \frac{\pi}{4})$$

$$y \approx 1.029204$$

Comparing this with the decimal approximation the calculator gives, it is pretty close.

$$\tan(0.8) \approx 1.0296356$$

Now You Try

1) Use the equation $f(x) = \sqrt[3]{x}$ at $x = 64$ to approximate $\sqrt[3]{60}$.

2) Use the equation $f(x) = \frac{1}{x^2}$ at $x = 5$ to approximate $\frac{1}{24}$.

3) Use the equation $f(x) = \sin(x)$ at $x = \pi$ to approximate $\sin(3)$.

Answers: 1) $3.91\overline{6}$ 2) 0.041616328 3) 0.141592653