Composite Functions

Given two functions, combine them in a way such that the outputs of one function become the inputs for the other, making it a composite function.

$$(f \circ g)(x) = f(g(x)) \quad \text{OR} \quad (f \circ g)(x) = f \text{ "composed of" } g$$

Evaluating Composite Functions

Evaluate the function on the right side, and then substitute that result into the other function to find the answer.

Example: Given $f(x) = 5x - 3$ and $g(x) = x^2$, find $(f \circ g)(3)$.

Solutions:

Step 1: Set up the equation and start from the right side.

$$(f \circ g)(3) = f(g(3))$$

Notice (3) g is the input for $f(x)$, so start by solving for g(3).

Given $g(x) = x^2$:
$g(3) = (3)^2$
$g(3) = 9$

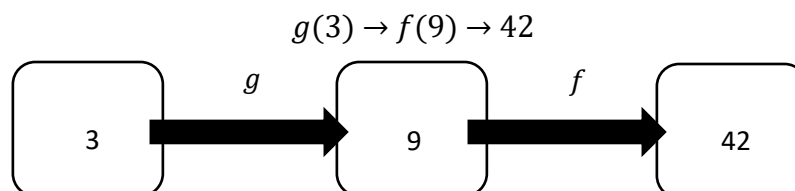**Step 2:** Now substitute the answer for $g(3)$ into $f(x)$.

$$f(g(3)) = f(9)$$

Given $f(x) = 5x - 3$:
$f(9) = 5(9) - 3$
$f(9) = 42$

so $(f \circ g)(3) = \mathbf{42}$

$$g(3) \rightarrow f(9) \rightarrow 42$$

| 3 | $\xrightarrow{g}$ | 9 | $\xrightarrow{f}$ | 42 |

Notice $f(x)$ is the input for $g(x)$, $g(x^2 + 4) =$ so start $\dfrac{x}{(x^2 + 4)}$ with $f(x)$.

Given $f(x) = x^2 + 4$:

$g(f(x)) = g(x^2 + 4)$ so $g(f(x)) = \dfrac{1}{x}{}^2 + 4$

Example: Given $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$ find $f(g(x))$.

Solution:

Since $f(g(x))$ uses $g(x)$ as the input for f, substitute $x + 1$ for $g(x)$ and simplify.

Finding the Composite Function

To compose two functions, redefine the composition by using the definition to find $f(g(x))$ or $g(f(x))$.

Example: Given $f(x) = x^2 + 4$ and $g(x) = \frac{1}{x}$, find $(g \circ f)(x)$.

Solution:

Step 1: Set up the function using the definition.

$(g \circ f)(x) = g(f(x))$
Step 1: Substitute.
$f(g(x)) = f(x + 1)$
$f(x + 1) = (x + 1)^2 + 2(x + 1) - 3$

Step 2: Now substitute $x^2 + 4$ into $g(x)$ for every x. Simplify as needed.

Given $g(x)$ $\dfrac{1}{x}$ $=:$
Step 2: Simplify.
$f(x + 1) = (x^2 + 2x + 1) + 2x + 2 - 3$
$f(x + 1) = x^2 + 4x$

Practice Exercises:
1. Given $f(x) = 2x - 6$ and $g(x) = x^2 + 3$, find $g(f(x))$.
2. Given $f(x) = 4 - x$ and $g(x) = x^3 - 1$, find $(f \circ g)(x)$.
3. Given $f(x) = 3x + 4$ and $g(x) = 2x$, find $(f \circ g)(5)$.
4. Given $f(x) = x + 7$ and $g(x) = \frac{1}{x^2 - 1}$ find $g(f(2))$.

Answers:
1. $g(f(x)) = 4x^2 - 24x + 39$ 3. $f(g(5)) = 34$

2. $f(g(x)) = 5 - x^3$ 4. $g(f(2)) = \frac{1}{80}$