Slope–Intercept Form and Point–Slope Form

Slope of the line	$m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$
Slope–Intercept Form	$y = mx + b;$ where m is slope and b is the y–intercept
Point–Slope Form	$y - y_1 = m(x - x_1)$ or $y = m(x - x_1) + y_1$
Slope of parallel lines	$m_1 = m_2$ (slopes are the same)
Slope of perpendicular lines	$m_1 = -\dfrac{1}{m_2}$ (slopes are opposite & reciprocal)
Equations of Horizontal and Vertical Lines	Horizontal: $y = b$ Vertical: $x = a;$ where a is the x intercept

Example (1): Write the slope – intercept equation of a line which passes through $(0,-7)$ whose slope is 2.

Solution: Slope–intercept equation is $y = mx + b$ **.** What we need to complete this equation are <u>slope</u> (m) & <u>y–intercept</u> (b), and the problem provides both parts.

$m = 2$ and $b = -7$ The equation of the line is $\boldsymbol{y = 2x - 7}$

Example (2): Write the slope–intercept equation of a line which passes through $(0,4)$ and $(3,-5)$.

Solution: Slope–intercept equation is $y = mx + b$ What we need to complete this equation are <u>slope</u> (m) & <u>y–intercept</u> (b), however, we only have <u>y–intercept</u>. To find the slope,

$$m = \frac{\text{rise}}{\text{run}} = \frac{-5 - 4}{3 - 0} = \frac{-9}{3} = -3$$

$m = -3$ and $b = 4$ The equation of the line is $\boldsymbol{y = -3x + 4}$

Example (3): Write the slope–intercept equation of a line which passes through $(-1,4)$ whose slope is 5.

Solution: Slope–intercept equation is $y = mx + b$ **.** What we need to complete this equation are <u>slope</u> (m) & <u>y-intercept</u> (b), however, we only have <u>slope</u>. Here there are two ways to find the equation of the line.

Method I We will substitute m and (x_1, y_1) into the equation $y = mx + b$.

$$m = 5 \text{ and } (x_1, y_1) = (-1,4)$$
$$4 = 5(-1) + b$$
$$4 = -5 + b$$
$$9 = b$$
$$\boldsymbol{y = 5x + 9}$$

Method II Since we are given slope m and an ordered pair (x_1, y_1), we can use Point–slope form $y - y_1 = m(x - x_1)$ to find equation of the line.

$$m = 5 \text{ and } (x_1, y_1) = (-1,4)$$
$$y - 4 = 5(x - -1)$$
$$y - 4 = 5(x + 1)$$
$$y - 4 = 5x + 5$$
$$y = 5x + 5 + 4$$
$$\boldsymbol{y = 5x + 9}$$

Example (4): Write the slope–intercept equation of a line which passes through $(1,3)$ and $(-5, -1)$

Solution: Slope–intercept equation is $y = mx + b$. What we need to complete this equation are <u>slope</u> & <u>y-intercept</u>. However, we are given two ordered pairs (x_1, y_1) and (x_2, y_2) without <u>slope</u> or <u>y-intercept</u>. Therefore, we need to find the <u>slope</u> first. Then we can use the two methods discussed on Example (3) to find the equation of the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-1 - 3}{-5 - 1} = \frac{-4}{-6} = \frac{2}{3}$$

$\boxed{\text{Method I}}$ Now we have slope, we will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$$y = mx + b$$
$$3 = \frac{2}{3}(1) + b$$
$$3 = \frac{2}{3} + b$$
$$3 - \frac{2}{3} = b$$
$$\frac{9}{3} - \frac{2}{3} = b$$
$$\frac{7}{3} = b$$
$$\boldsymbol{y = \frac{2}{3}x + \frac{7}{3}}$$

$\boxed{\text{Method II}}$ We also can use Point-slope form to find the equation of the line.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{2}{3}(x - 1)$$
$$y - 3 = \frac{2}{3}x - \frac{2}{3}$$
$$y = \frac{2}{3}x - \frac{2}{3} + 3$$
$$\boldsymbol{y = \frac{2}{3}x + \frac{7}{3}}$$

Example (5): Write the equations of a parallel and perpendicular line to the equation: $y = 2x + 1$.

Parallel: Parallel lines have equal slopes, so any line with $m = 2$ is parallel.
Perpendicular: Perpendicular lines have a negative reciprocal slope. Any line with $m = -\frac{1}{2}$ will be perpendicular.

Example (6): Write the equations of a vertical line and a horizontal line passing through (3,4).

Vertical: $x = 3$
Horizontal: $y = 4$

Exercises:

1. Write the slope – intercept equation of a line which passes through $(1,3)$ whose slope is 4.
2. Write the slope–intercept equation of a line which passes through $(1,2)$ and $(5,3)$.
3. Write the slope–intercept equation of a line which is parallel to $y = 4x + 1$, passing through $(8,2)$.
4. Write the slope–intercept equation of a line which is perpendicular to $y = -x + 2$, passing through $(-1,1)$.
5. Write an equation of a horizontal line which passes through $(-2, -4)$.
6. Write an equation of a vertical line which passes through $(-5,6)$.

Answers:

1. $y = 4x - 1$
2. $y = \frac{1}{4}x + \frac{7}{4}$
3. $y = 4x - 30$
4. $y = x + 2$
5. $y = -4$
6. $x = -5$