Solving Linear Inequalities and Compound Inequalities

    Steps for solving linear inequalities are very similar to the steps for solving linear equations. The big differences are multiplying and dividing a constant on the inequalities and expressing the solution set. This handout will show some examples on how to solve linear inequalities and compound inequalities and how to express the solution sets of inequalities.

Solve Linear Inequalities

Example (1):

$$3x - 5 < 7$$
$$3x - 5 + 5 < 7 + 5$$
$$3x < 12$$
$$\frac{3}{3}x < \frac{12}{3}$$
$$\boldsymbol{x < 4}$$

Example (2):

$$\frac{2x + 1}{4} \geq 3$$
$$4 * \frac{2x + 1}{4} \geq 3 * 4$$
$$2x + 1 \geq 12$$
$$2x + 1 - 1 \geq 12 - 1$$
$$2x \geq 11$$
$$\frac{2}{2}x \geq \frac{11}{2}$$
$$x \geq \frac{11}{2}$$

Example (3):

$$5 - 2(x + 3) \leq 3x - 7$$
$$5 - 2(x + 3) \leq 3x - 7$$
$$5 - 2x - 6 \leq 3x - 7$$
$$-2x - 1 \leq 3x - 7$$
$$+2x - 2x - 1 \leq 3x - 7 + 2x$$
$$-1 \leq 5x - 7$$
$$+7 - 1 \leq 5x - 7 + 7$$
$$6 \leq 5x$$

$$\frac{6}{5} \leq x$$

$$x \geq \frac{6}{5}$$

Example (4):

$$|2x - 4| \leq 6$$

This splits into two equations:

$$2x - 4 \leq 6 \qquad\qquad 2x - 4 \geq -6$$
$$2x - 4 + 4 \leq 6 + 4 \qquad 2x - 4 + 4 \geq -6 + 4$$
$$2x \leq 10 \qquad 2x \geq -2$$
$$x \leq 5 \qquad x \geq -1$$

Compound inequalities like this can also be written and solved like this:
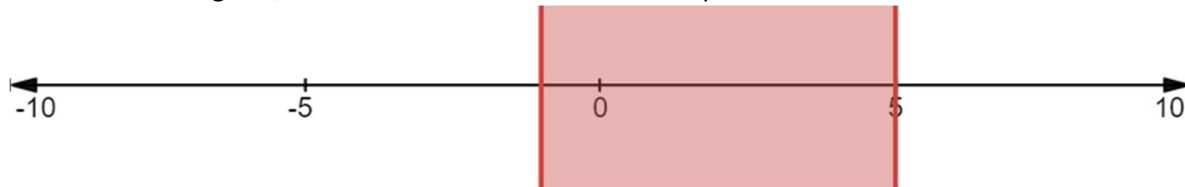
$$-6 \leq 2x - 4 \leq 6$$
$$-6 + 4 \leq 2x - 4 + 4 \leq 6 + 4$$
$$-2 \leq 2x \leq 10$$
$$-\frac{2}{2} \leq \frac{2}{2}x \leq \frac{10}{2}$$
$$-1 \leq x \leq 5$$

When considering this, feel free to use a number line to represent the values

Example (5):

$$\frac{3}{x - 1} > 2$$
$$(x - 1) * \frac{3}{x - 1} > 2 * (x - 1)$$
$$3 > 2x - 2$$
$$3 + 2 > 2x - 2 + 2$$
$$5 > 2x$$
$$\frac{5}{2} > x$$
$$x < \frac{5}{2}$$

Example (6): When you multiply or divide by a negative, you must flip the sign

$$-2x > 4$$
$$x < -2$$