# Adding Fractions and Mixed Numbers

I. Adding Fractions with the Same Denominator

Once you have identified the denominator is the same, you mainly focus on the numerator.

**Example:**

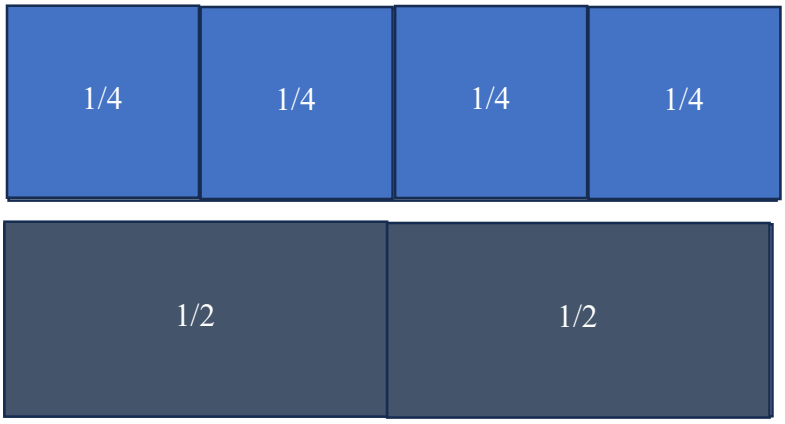
What is ¼ + ¼?

$$\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$$

Initially, the denominator remains the same, however the fraction can be reduced. This is called writing the fraction in simplest form.

$$\frac{2}{4}=\frac{1}{2}$$

This can also be visualized like so:




**Note:** The denominator determines how many parts make up the whole. The numerator tells you how many parts to use. Which is why $\frac{2}{4}=\frac{1}{2}$. 2 of the ¼ pieces are equal to 1 of the ½ pieces.

II. Adding Fractions with Unlike Denominators

This raises an issue when we are adding fractions with unlike denominators. If I was solving a problem like $\frac{1}{3}+\frac{2}{5}$, it would not make sense to try to see how to combine 1 1/3 piece and 2 1/5 pieces. To solve this issue we simply find the LCD (Least Common Denominator).

**Example:** $\frac{1}{3}+\frac{2}{5}$

Compare the multiples of 3 and 5 to find the smallest number they share.

$$3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36$$
$$5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60$$

15 and 30 are the visible common multiples. It is important to note there is an infinite amount of share multiples, therefore it is easiest to take the SMALLEST shared multiple. It is also important to note that 15 is simply $5 * 3$. This is a method to find a common multiple, but it is not guaranteed to be the SMALLEST common multiple.

To change the denominators to 15, we must multiply the fraction by the number it takes to make the denominator 15 in the form of $\frac{x}{x}$. Why do we do it this way? When multiplying a fraction, you can multiply by any number over itself, as that is the fractional form of multiplying by 1. $\ldots\frac{5}{5}, \frac{4}{4}, \frac{3}{3}\ldots = 1.$

$$\frac{1}{3} * \frac{5}{5} = \frac{5}{15}$$
$$\frac{2}{5} * \frac{3}{3} = \frac{6}{15}$$

Now we can follow the same rules from above, adding the numerators, leaving the denominators alone, then checking for simplest form.

$$\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

This is in simplest form as there are no shared factors of 11 and 15.

III. Adding Mixed Numbers and Fractions

Sometimes fractions that are greater than 1 are written as mixed numbers like 1 ¾ cup or 2 ½ lbs. These mixed numbers can make addition exceptionally difficult. So, just like we did when adding numbers with different denominators, we will have to convert the fraction to a form we can use. This form is called <u>improper fraction</u> form.

**Example:**

$$1\frac{3}{4}$$

What we are going to do to convert this to an improper fraction is multiply the constant term (1) by the denominator (4) and then add it to the numerator (3).

$$(1 * 4) + 3 = 7 = \text{numerator}$$

New fraction: $\frac{7}{4}$

**Note:** The denominator does NOT change with this procedure.

**Example:**

$$2\frac{3}{5} + 4\frac{1}{7}$$

First convert them both to improper fractions

$$(2 * 5) + 3 = 13 \qquad (4 * 7) + 1 = 29$$

$$\frac{13}{5} + \frac{29}{7}$$

The denominators are not the same, so we must find common multiples

5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50

7: 7, 14, 21, 28, 35, 42, 49, 56,

35 is the least common multiple so it will be the common denominator

$$\left(\frac{13}{5} \cdot \frac{7}{7}\right) + \left(\frac{29}{7} \cdot \frac{5}{5}\right)$$

$$\frac{91}{35} + \frac{145}{35}$$

$$\frac{236}{35} = 6\frac{25}{35} = 6\frac{5}{7}$$