Simplifying Square Roots (extended version)

The square root of any positive number is the number that can be squared to get the number whose square root we are seeking. For example,

$\sqrt{16} = 4$ because if we square 4 we get 16, which is the number whose square root is being found.

$\sqrt{64} = 8$ because $8^2 = 64$

The symbol $\sqrt{}$ is called a radical and it is read as "the square root of."

The number underneath the radical is called the "radicand." In the expression $\sqrt{36}$, the radicand is 36.

It should be noted that each positive number has 2 square roots. One is the positive or *principal* square root, and the other is the *negative* square root.

$\sqrt{4} = 2, -2$ because $2^2 = 4$ & $(-2)^2 = 4$

Note that we cannot have a negative sign under the radical if we are limited to real solutions.

$\sqrt{-4}$ is not a real number, because there is no number that we can multiply by itself and get -4.

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To simplify a radical, we must look for and remove any perfect square factors that may be in the radicand. REMEMBER that a perfect square is the square of an integer.

$$3^2 = 9$$
$$4^2 = 16$$
$$8^2 = 64$$

A radical expression is in simplest form if the radicand contains no perfect square factors. To simplify a radical, we will first find the prime factorization of the radicand and rewrite the radicand in exponential form.

$64 = 2^6$ prime factorizations of 64 in exponential form.

If the exponent is an even number, then the number itself is a perfect square.

To take the square root of 2^6 we <u>remove the radical and divide the exponent by 2.</u>

EXAMPLE:

$\sqrt{64} = \sqrt{2^6} = 2^{6/2} = 2^3 = 8$

Once we have divided the exponent by 2, we can multiply out the remaining factors.

EXAMPLES:

$$\sqrt{81} = \sqrt{3^4} = 3^{4/2} = 3^2 = 9$$
$$\sqrt{196} = \sqrt{2^2 \cdot 7^2} = 2^{2/2} \cdot 7^{2/2} = 2 \cdot 7 = 14$$

Often the number we wish to simplify is not a perfect square. We then must find any perfect square factors contained in the number and remove them from under the radical by taking their square roots. To simplify $\sqrt{40}$, first find the prime factorization of 40.

NOTICE that the exponents are odd numbers. This means that 2^3 and 5^1 are <u>not</u> perfect squares.

- Any prime factor with an exponent of 1 will not be a perfect square nor will it contain a perfect square.
- Any prime factor with an <u>even</u> exponent will be a perfect square.
- Any prime factor with an <u>odd</u> exponent of 3 or higher will <u>contain</u> a perfect square factor.

$$\sqrt{40} = \sqrt{2^3 \cdot 5}$$
$$\sqrt{40} = \sqrt{2^2 \cdot 2^1 \cdot 5}$$

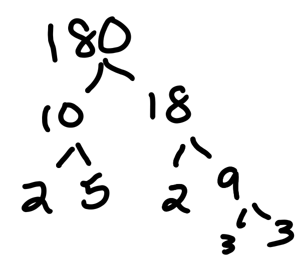
Separate any perfect squares. The Product Property of Square Roots allows us to rewrite a product under a radical as a product of 2 separate radicals

We now have one radical which is a perfect square and one which is not. We can take the square root of the perfect square and multiply the factors remaining under the radical.

$$\sqrt{2^2} \cdot \sqrt{2 \cdot 5}$$
$$2\sqrt{10}$$

EXAMPLE: Simplify $5\sqrt{180}$. Notice that this is "5 times the square root of 180." We must simplify the $\sqrt{180}$ first and then multiply by 5.

$5\sqrt{180} = 5\sqrt{2^2 \cdot 3^2 \cdot 5}$	Find the Prime factorization of 180.
$= 5\sqrt{2^2}\sqrt{3^2}\sqrt{5}$	Separate the perfect squares.
$= 5 \cdot 2^{\frac{2}{2}} \cdot 3^{\frac{2}{2}} \cdot \sqrt{5}$	Simplify.
$= 5 \cdot 2 \cdot 3\sqrt{5}$	Multiply.
$= 30\sqrt{5}$	



$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$180 = 2^2 \cdot 3^2 \cdot 5$

Many of the expressions we will need to simplify will contain variables. The same process applies.

$$\sqrt{x^6} = x^{6/2} = x^3$$
$$\sqrt{x^9} = \sqrt{x^8 \cdot x} = x^{8/2}\sqrt{x} = x\sqrt[4]{x}$$

Any variable radical expression which has an exponent of 1 will not be a perfect square nor will it contain a perfect square factor. Any variable expression which has an odd exponent of 3 or higher will contain a perfect square factor.

$$\sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2}\sqrt{x} = x\sqrt{x}$$

We often have radicals which have both numbers and variables.

EXAMPLES: $\sqrt{45x^2y^3}$

$\sqrt{45x^2y^3} = \sqrt{3^2 \cdot 5 \cdot x^2 \cdot y^2 \cdot y}$	Find the prime factorization of 45.
$=\sqrt{3^2}\sqrt{x^2}\sqrt{y^2}\sqrt{5y}$	Separate the perfect squares and take the roots
$= 3xy\sqrt{5y}$	Multiply the numbers and multiply the variables

Exercises. Simplify each of the following:

1 · $\sqrt{49}$

2 · $\sqrt{12}$

3 · $\sqrt{40}$

4 · $\sqrt{y^{15}}$

5. $\sqrt{24x^2y^5}$

6. $3\sqrt{20a^5b^2}$

7. $2xy\sqrt{120x^5y^3}$

8. $2\sqrt{64x^8y^{10}}$

KEY

1. 7	2. $2\sqrt{3}$	3. $2\sqrt{10}$	4. $y^7\sqrt{y}$
5. $2xy\sqrt[2]{6y}$	6. $6a^2b\sqrt{5a}$	7. $4x^3y\sqrt[2]{30xy}$	8. $16x^4y^5$