Permutations

The permutation method of counting is used when order is important (stated or implied) and repetition is not allowed. For example, when we are counting the number ways different prizes can be awarded, creating ID numbers with nonrepeating digits or license plates of nonrepeating letters and digits, we are counting the number of permutations.

The formula for the number of permutations of n things taken r at a time follows:

$$ {}_{n}P_{r} = \frac{n!}{(n-r)!} $$

Example 1

From a set of 10 different entries (persons or objects) in a contest, 3 are to be selected to receive first, second and third prize. In how many different ways can the prizes be awarded?

Order is important since the three entries chosen will be designated 1st, 2nd and 3rd. Repetition is not allowed since no entry can win more than one prize.

Solution method #1 using the Permutation Formula

$$ {}_{10}P_{3} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10\cdot9\cdot8\cdot\cancel{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}}{\cancel{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}} = \frac{720}{1} = 720 $$

10 entries taken 3 at a time where order is important and repetition is not allowed.

You can award 1st, 2nd and 3rd prize seven hundred twenty different ways.

Solution method #2 using the Fundamental Counting Principle

First place		Second place		Third place		
10	•	9	•	8	=	720
choices		choices		choices		different ways to award

The number of choices decreases because no single entry can be awarded more than one prize.

Example 2

How many different 5-digit identification numbers are possible if no digit is repeated?

Order is important since the same 5 digits used in one ID number can be repositioned to create a different ID number. For example, 57213 and 21573.

Repetition is not allowed as stated – “no digit is repeated”.

Solution method #1 using the Permutation Formula

$$_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!} = \frac{10\cdot9\cdot8\cdot7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}}{\cancel{5\cdot4\cdot3\cdot2\cdot1}} = \frac{30240}{1} = 30240$$

Thirty thousand two hundred forty different ID numbers are possible.

Solution method #2 using the Fundamental Counting Principle

1^{st} digit		2nd digit		3rd digit		4th digit		5th digit		
10	•	9	•	8	•	7	•	6	=	30240
number of choices		number of choices		number of choices		number of choices		number of choices		possible ID numbers

Example 3

How many different license plates are possible if the plates consist of 3 letters followed by 3 digits and repetition of letters or digits is not allowed?

Solution using the Permutation Formula

$$_{26}P_3 \bullet {}_{10}P_3 = \frac{26!}{(26-3)!} \bullet \frac{10!}{(10-3)!}$$

$$= \frac{26!}{23!} \bullet \frac{10!}{7!}$$

$$= 26 \cdot 25 \cdot 24 \bullet 10 \cdot 9 \cdot 8$$

$$= 11{,}232{,}000$$

There are eleven million two hundred thirty-two thousand different license plates possible.