Changing Between Base Two and Base Ten

Base Ten

Base 10 consists of 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The position of the digit determines its value.

Consider the different positions and values for the digit 3 in the following numbers

2173 here the 3 means: 3 times 1 or 3

2137 here the 3 means: 3 times 10 or 30

2317 here the 3 means: 3 times 100 or 300

3171 here the 3 means: 3 times 1000 or 3000

Standard Form: The number 64,702 is in standard form. its base ten values are:

6	4	7	0	2
10^4	10^3	10^2	10^1	10^0
10000	1000	100	10	1

Expanded Form: The same number in expanded form is;

$$(6 \times 10^4) + (4 \times 10^3) + (7 \times 10^2) + (0 \times 10^1) + (2 \times 10^0)$$
$$60000 \quad + \quad 4000 \quad + \quad 700 \quad + \quad 0 \quad + \quad 2$$

Base Two

Base two consists of two digits 0 and 1. Like base 10 its value is determined by its position

Standard Form: The number 1 1 0 1 1 is in standard form. Its base two places are:

1	1	0	1	1 TWO
2^4	2^3	2^2	2^1	2^0
16	8	4	2	1

Expanded Form: The same number in expanded form is:

$$\begin{aligned} 11011_{\text{TWO}} &= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= 16 + 8 + 0 + 2 + 1 \\ &= 27_{\text{TEN}} \end{aligned}$$

Changing base two to ten

Example 1. Write 101111_{TWO} as a base ten number

<u>Solution</u>: $101111_{TWO} = (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= 32 + 0 + 8 + 4 + 2 + 1$

$= 47_{TEN}$

Example 2. An alternative method: Write 101111_{TWO} as a base ten number.

Step 1. Draw as many blanks as there are digits in the given base TWO number in this example there are 6.

______ ______ ______ ______ ______ ______ TWO

Step 2. Beginning under the rightmost blank, label the base TWO place values, i.e., 1, 2, 4, 8, 16, 32.

______	______	______	______	______	______	TWO
32	16	8	4	2	1	

Step 3. Write the base TWO number in the blanks.

1	0	1	1	1	1	TWO
32	16	**8**	**4**	**2**	**1**	

Step 4. Add the place values under the 1s in the base TWO number:

32+8+4+2+1=47

Changing Base Ten to base two

Example 3: Write 365 as a base two number

Step 1. Make a guess of how many blanks you may need, you will adjust how many you have in step 3.

______ ______ ______ ______ ______ ______ ______ ______ ______ ______ TWO

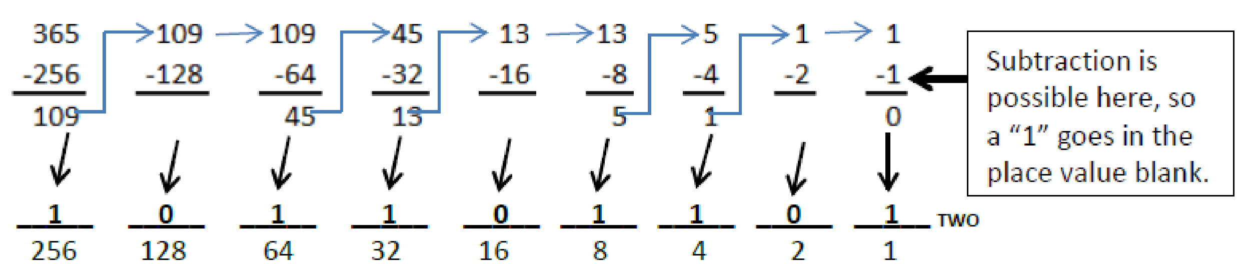
Step 2. Beginning under the rightmost blank, label the base two values. i.e. 1,2,4,8,16,32,64,132 and so on. Stop when the place values exceeds the given base ten number, which in this example is 365.

______	______	______	______	______	______	______	______	______	______	______	TWO
	512	256	128	64	32	16	8	4	2	1	

Step 3. Erase the blanks and place values that exceed the given number.

______	______	______	______	______	______	______	______	______	TWO
256	128	64	32	16	8	4	2	1	

Step 4. Beginning with the given number, subtract the largest place value and put a 1 in that place value blank. Use the difference and try to subtract the next place value. If the subtraction is not possible without getting a negative number, put a 0 in the place value blank and try the next place value. Continue in this manner putting a 1 in each of the place value blanks where the subtraction is possible and a 0 in the blanks where the subtraction is not possible.



Step 5. Use the method in example 2 to check your work