A one-tailed Hypothesis Test of a Proportion

In testing a Hypothesis about a population proportion, there are FIVE steps:

1. Identify the claim and Hypotheses
2. Information and Test Statistic.
3. Find the p-value
4. Interpret Test Results
5. Write the Conclusion

Identify the Claim and write the Null Hypothesis (H_0) and the Alternative Hypothesis (H_1).

Example: Medics and teachers believe that a new vitamin supplement will help decrease the number of students absent due to sickness during the winter. They took a sample of 742 students. They gave the vitamin supplement to the students for the months of August through December, and observed about 8% of the students were absent due to sickness. Historically, students have been absent about 10% of the time due to illness. Is the decrease significantly large enough (significance level = 0.05) to conclude that the vitamin supplement reduces absenteeism due to sickness?

H_0: p = 0.10, this is the usual proportion of absentees.

H_1: p < 0.10, teachers and medics believe (i.e., claim) the supplement will decrease this proportion.

Identify the information and calculate the test statistic.

For this example:

Population Proportion: p=0.01 Significance Level = 0.05.

The test statistic is:

$$Z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \quad Z = \frac{0.08-0.10}{\sqrt{\frac{0.10(1-0.10)}{742}}} \quad Z = -1.815978463$$

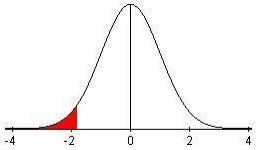
Find the p-value, begin by considering the Standard Normal Distribution.

This Hypothesis Test is a one-tailed (left-tail) test because H_0 will only be rejected in favor of H_1 if the test statistic is significantly less than the mean. Notice that the inequality symbol, **<**, in the Alternative Hypothesis points in the direction of the tail.

The Test Statistic, Z = -1815978463. For a left-tail test, the p-value is the area under the curve to the left of the test statistic – the shaded area on the drawing.

To find the p-value, using the **normalcdf** function on the calculator:

2^{nd} VARS > 2: normalcdf > ENTER: normalcdf (left bound, right bound, mean, standard deviation): normalcdf (-E99,-1.815978463,0,1) = 0.0346867815 ≈ 0.035



Interpreting the Test results. Compare the P-value with the Significance Level = 0.05.

The p-value of 0.035 is less than the Significance Level α=0.05 so the decision is to reject the Null Hypothesis. Because H_0 is rejected, the evidence points to the Alternative Hypothesis, H_1. Therefore, there is evidence to support the claim.

Conclusion: Write the conclusion in English in the context of the problem.

The belief held by the medics and teachers is valid; administrating the vitamin supplement significantly decreases the absentees due to sickness.

With the Texas Instruments calculator:

Example:

Press STAT scroll to TESTS select option 5: 1-PropZTest press ENTER

This is the calculator input:	**This is the calculator output:**
p_0: 0.1	1-PropZTest
X: (.08)(742) = 59.36 (round to 59 the nearest whole number or you will get an message.) **p = .0314404472**	prop < .1
n: 742 p-hat = .0795148248 prop: < p_0	Z = -1.860031849 error
Calculate	n = 742

When using the calculator, both the test statistic and the p-value are different from “by hand” due to the rounding done (59.36 ≈ 59). However the conclusion, based on the p- value, is the same.