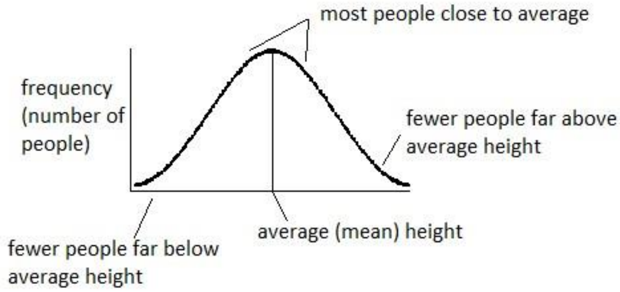
Z-scores and the Empirical Rule

The heights of individuals within a certain population are normally distributed. That is, it will be found that most peoples' heights are clustered around an average height, and the farther a particular height is above or below the average, the less people we would find with that height. Graphically, it looks like this:



A Z-score tells how many Standard Deviations a value is <u>above</u> or <u>below</u> the Mean.

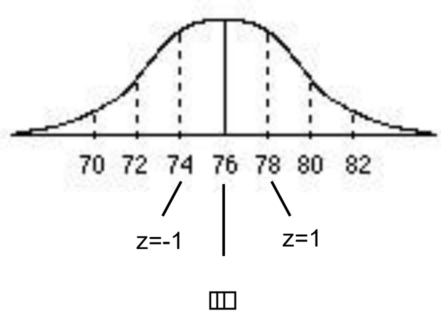
Suppose that the Mean height (μ) of college basketball players is 6 feet 4 inches (chance the feet to inches; only one measuring unit is needed 76 inches = μ) with a Standard Deviation (σ) of 2 inches.

- A height of 78" (X) is 2" <u>above</u> the Mean; 2" is one Standard Deviation; so the Z-score associated with 78" is positive 1.

$$\frac{X-\mu}{\sigma} = \frac{78-76}{2} = \frac{2}{2} = 1$$

- A height of 74" (X) is 2" <u>below</u> the Mean; 2" one Standard Deviation; so the Z-score associated with 72" is <u>negative</u> 1.

$$\frac{X-\mu}{\sigma} = \frac{74-76}{2} = -\frac{2}{2} = -1$$



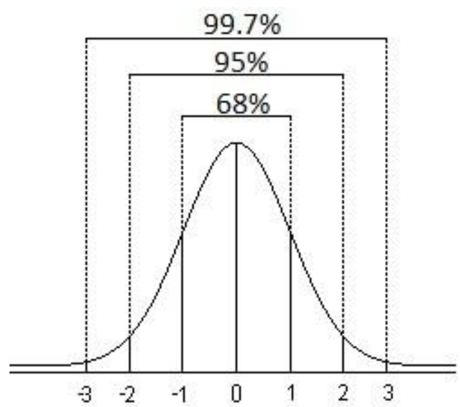
Empirical Rule or 68-95-99.7 Rule:

The Empirical Rule says that:

- Approximately 68% of the distribution will be within one standard deviation of the mean. This is $Z = \pm 1$

- Approximately 95% of the distribution will be within two standard deviations of the mean. This is $Z = \pm 2$

- Approximately 99.7% of the distribution will be within three standard deviations of the mean. This is $Z = \pm 3$



It is easier to work with the Empirical Rule if the percentages are broken down evenly. The 68% can be split into 34% on each side of the Mean, so from the Mean to the First Z-score there will be 34% of the Distribution.

This can also be applied to the 95%. If it is split in half, there will be 47.5% between the Mean and the Second Z-score. To calculate the percentage between the First Z-score and the Second Z-score just subtract the 34% from the 47.5%: 47.5%-34%=13.5%

This can be done to the Third Z-score and the 99.7%. The result of the breakdown will look like this: