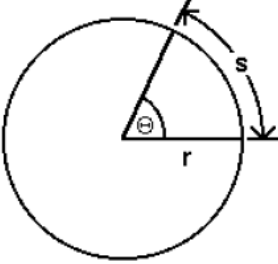


Finding Angular and Linear Speeds by Dimensional Analysis i.e. Unit Conversion

<p>Definitions of Symbols</p> <p>C = circumference r = radius d = diameter (d = 2r) s = arc length Θ = central angle <i>in radians</i> π = approximately 3.14</p> <p>Formulas</p> $C = 2\pi r = \pi d \text{ and } \Theta = \frac{s}{r}$		<p>Relationships</p> <p>1 revolution = 1 turn around circle (definition)</p> <p>1 revolution = 2π radians (angular measurement)</p> <p>1 revolution = $2\pi r$ (linear measurement, i.e. distance)</p>
NOTES:	when $s = r$, $\Theta = 1$ radian	when $s = C$, $\Theta = 2\pi$ radians

EXAMPLES:

(Note CANCELLATION OF UNITS! Un-cancelled units are **boldfaced**.)

1. A phonograph record has a radius of 3 inches and revolves at 45 RPM. Find the linear speed of the outside edge of the record.

Solution: Use the fact that 1 revolution = $2\pi r$:

$$\left(\frac{45 \text{ revolutions}}{1 \text{ minute}} \right) = \left(\frac{45 \text{ revolutions}}{1 \text{ minute}} \right) \cdot \left(\frac{2\pi(3) \text{ inches}}{1 \text{ revolution}} \right) = (848.2 \frac{\text{inches}}{\text{minute}})$$

2. A car is traveling 60 mph. The diameter of the wheels is 3 ft.
 - a. Find the number of revolutions per minute the wheels are rotating.

Strategy: We need to convert $\frac{\text{mile}}{\text{hour}} \rightarrow \frac{\text{feet}}{\text{hour}} \rightarrow \frac{\text{feet}}{\text{minute}} \rightarrow \frac{\text{revolutions}}{\text{minute}}$
so, we need three conversion ratios.

$$\left(60 \frac{\text{miles}}{\text{hour}} \right) = \left(60 \frac{\text{miles}}{\text{hour}} \right) \cdot \left(\frac{5280 \text{ feet}}{1 \text{ mile}} \right) \quad (\text{since } 1 \text{ mile} = 5280 \text{ feet})$$

$$\left(60 \frac{\text{miles}}{\text{hour}} \right) \cdot \left(\frac{5280 \text{ feet}}{1 \text{ mile}} \right) \cdot \left(\frac{1 \text{ hour}}{60 \text{ minutes}} \right) \quad (\text{since } 1 \text{ hour} = 60 \text{ minutes})$$

$$\left(60 \frac{\text{miles}}{\text{hour}} \right) \cdot \left(\frac{5280 \text{ feet}}{1 \text{ mile}} \right) \cdot \left(\frac{1 \text{ hour}}{60 \text{ minutes}} \right) \cdot \left(\frac{1 \text{ revolution}}{2\pi(1.5) \text{ feet}} \right) = \left(560.2 \frac{\text{revolutions}}{\text{minute}} \right)$$

- b. What's the angular speed of the wheels in radians per minute?

$$\left(560.2 \frac{\text{revolutions}}{\text{minute}} \right) \cdot \left(\frac{2\pi \text{ radians}}{1 \text{ revolution}} \right) = \left(3519.8 \frac{\text{radians}}{\text{minute}} \right) \quad (\text{since } 1 \text{ rev.} = 2\pi \text{ radians})$$