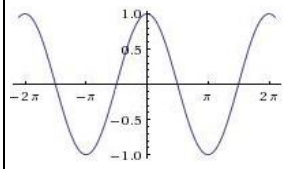
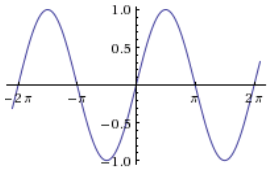
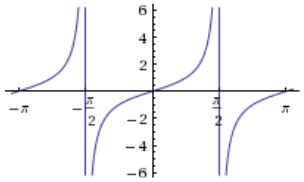
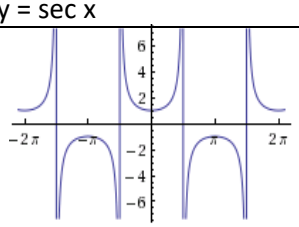
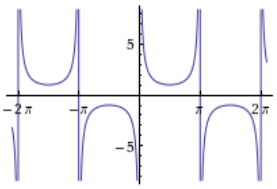
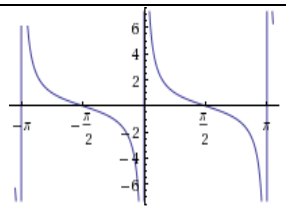
MAC 1114 – Trigonometry Basic Graphs

Basic Trigonometric Graphs:

y = cos x	y = sin x	y = tan x
		
y = sec x	**y = csc x**	**y = cot x**
		

Standard Forms

y = a sin k(x – b) +c y = a cos k(x – b) +c	Amplitude= a	Period = $\frac{2\pi}{k}$, k > 0	Phase shift: b	Vertical shift: c
y = a csc k(x – b) +c y = a sec k(x – b) +c	Not applicable	Period = $\frac{2\pi}{k}$, k > 0	Phase shift: b	Vertical shift: c
y = a tan k(x – b) +c y = a cot k(x – b) +c	Not applicable	Period = $\frac{\pi}{k}$, k > 0	Phase shift: b	Vertical shift: c

Examples (these show one period for each example)

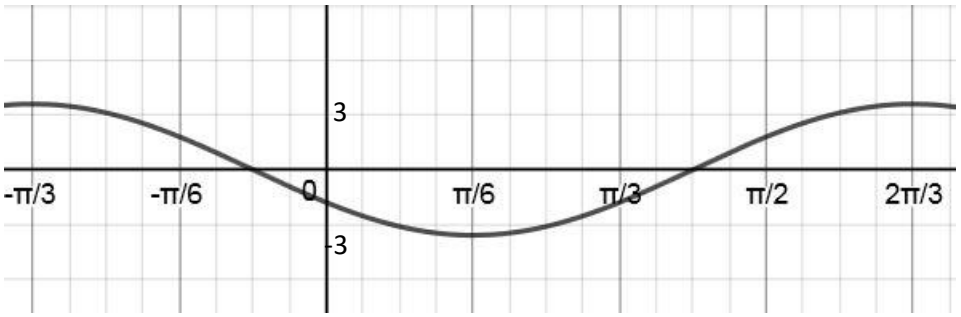
1. $\boldsymbol{y = 3\cos(2x + \frac{2\pi}{3})}$**:** put it into the standard form by factoring out the 2 that is with the x. This gives:
   $y = 3\cos 2(x + \frac{\pi}{3})$: Amplitude → 3,
   Period → $\frac{2\pi}{2} = \pi$ *so would divide graph into* $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ *and* π.
   Phase shift→$\frac{-\pi}{3}$ (to the left)

5 point method: (take the unshifted graph and adjust to get new points)

Starting point: unshifted (0,3) →$(0 - \frac{\pi}{3}, 3)$ Second point: unshifted $(\frac{\pi}{4}, 0)$ → $(\frac{\pi}{4} - \frac{\pi}{3}, 0)$

Third point: unshifted $\left(\frac{\pi}{2}, -3\right)$→ $(\frac{\pi}{2} - \frac{\pi}{3}, -3)$ Fourth point: unshifted $(\frac{3\pi}{4}, 0)$ → $(\frac{3\pi}{4} - \frac{\pi}{3}, 0)$

End of period: unshifted $(\pi, 3)$ → $(\pi - \frac{\pi}{3}, 3)$



angle	value
$\frac{-\pi}{3}$	3
$\frac{-\pi}{12}$	0
$\frac{\pi}{6}$	-3
$\frac{5\pi}{12}$	0
$\frac{2\pi}{3}$	3

2. $y = 2\csc\left(2x + \frac{\pi}{2}\right)$

put it into the standard form by factoring out the 2 that is with the x. This gives:

$y = 2\csc 2\left(x + \frac{\pi}{4}\right)$; period = $\frac{2\pi}{2} = \pi$ *so would divide graph into* $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ *and* π.

Phase shift→$\frac{-\pi}{4}$ (to the left)

5 point method: (take the unshifted graph and adjust to get new points)

Starting point: unshifted (0,1) → $(0 - \frac{\pi}{4}, 1)$

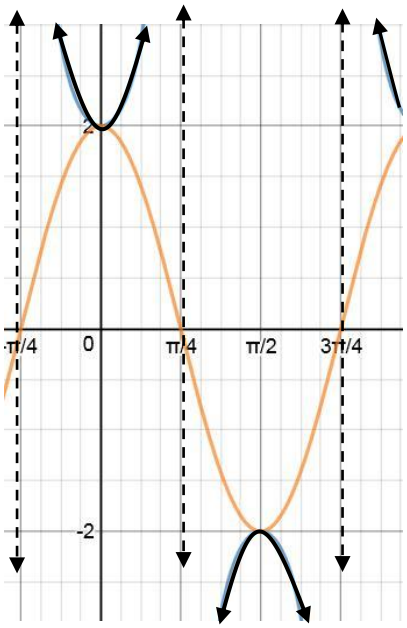
Second point: unshifted $(\frac{\pi}{4}, 0)$ → $(\frac{\pi}{4} - \frac{\pi}{4}, 0)$

Third point: unshifted $\left(\frac{\pi}{2}, -1\right)$→ $(\frac{\pi}{2} - \frac{\pi}{4}, 0)$

Fourth point: unshifted $(\frac{3\pi}{4}, 0)$ → $(\frac{3\pi}{4} - \frac{\pi}{4}, 0)$

End of period: unshifted $(\pi, 1)$ → $(\pi - \frac{\pi}{4}, 1)$

Hint: start with a sine graph with the same amplitude, shift and period, and the use it to graph it's inverse: the csc. The zero's become the asymptotes for the csc graph.



angle	value
$\frac{-\pi}{4}$	asymptote
0	2
$\frac{\pi}{4}$	asymptote
$\frac{\pi}{2}$	-2
$\frac{3\pi}{4}$	asymptote

3. $y = \tan\left(x - \frac{\pi}{4}\right)$

Period: no change since k = 1→ π so divide the graph into increments of $\frac{\pi}{4}$ like normal. Phase shift→ $\frac{\pi}{4}$

5 point method: (take the unshifted graph and adjust to get new points)

Starting point: unshifted $(\frac{-\pi}{2}, -\infty)$ → $(\frac{-\pi}{2} + \frac{\pi}{4}, -\infty)$

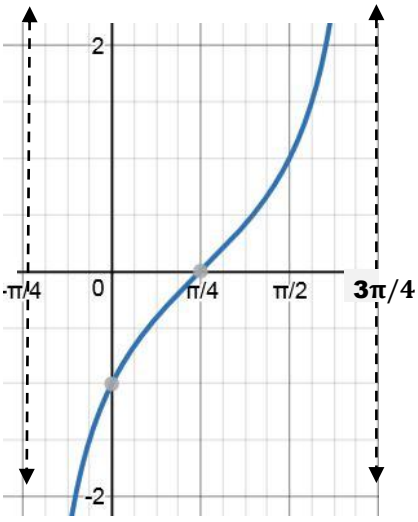
Second point: unshifted $(-\frac{\pi}{4}, -1)$ → $(-\frac{\pi}{4} + \frac{\pi}{4}, -1)$

Third point: unshifted $(0, 0)$ → $(0 + \frac{\pi}{4}, 0)$

Fourth point: unshifted $(\frac{\pi}{4}, 1)$ → $(\frac{\pi}{4} + \frac{\pi}{4}, 1)$

End of period: unshifted $(\frac{\pi}{2}, +\infty)$ → $(\frac{\pi}{2} + \frac{\pi}{4}, +\infty)$

angle	value
$\frac{-\pi}{4}$	$-\infty$
0	-1
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	1
$\frac{3\pi}{4}$	$+\infty$



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