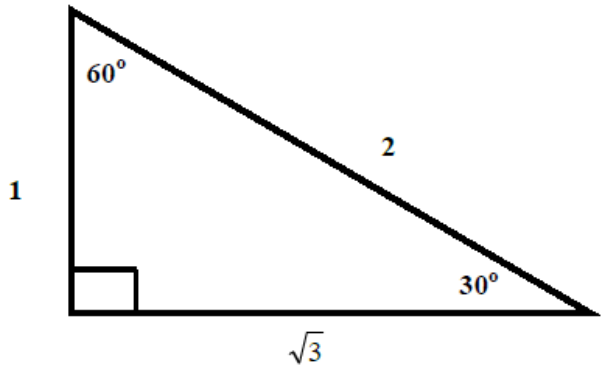
Trigonometry Fundamentals and the Unit Circle

The six trigonometric functions sine, cosine, tangent, cosecant, secant, cotangent are derived from relationships with right triangles. Common values result from two right triangles:

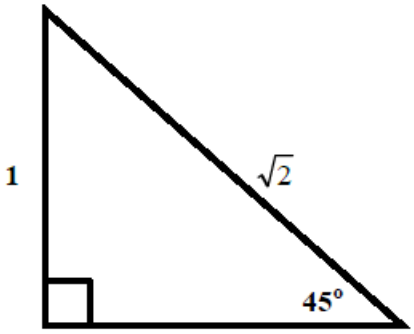


$$\sin(30°) = \frac{OPP}{HYP} = \frac{1}{2}$$

$$\cos(30°) = \frac{ADJ}{HYP} = \frac{\sqrt{3}}{2}$$

$$\tan(30°) = \frac{OPP}{ADJ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(Note: values for 60° are found similarly.)

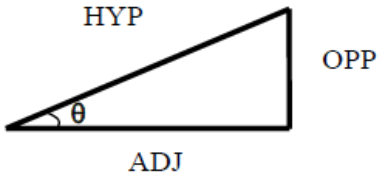


$$\sin(45°) = \frac{OPP}{HYP} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos(45°) = \frac{ADJ}{HYP} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan(45°) = \frac{OPP}{ADJ} = \frac{1}{1} = 1$$

"SOH CAH TOA"



SOH $\sin\theta = \frac{OPP}{HYP}$ $\quad \csc\theta = \frac{HYP}{OPP}$

CAH $\cos\theta = \frac{ADJ}{HYP}$ $\quad \sec\theta = \frac{HYP}{ADJ}$

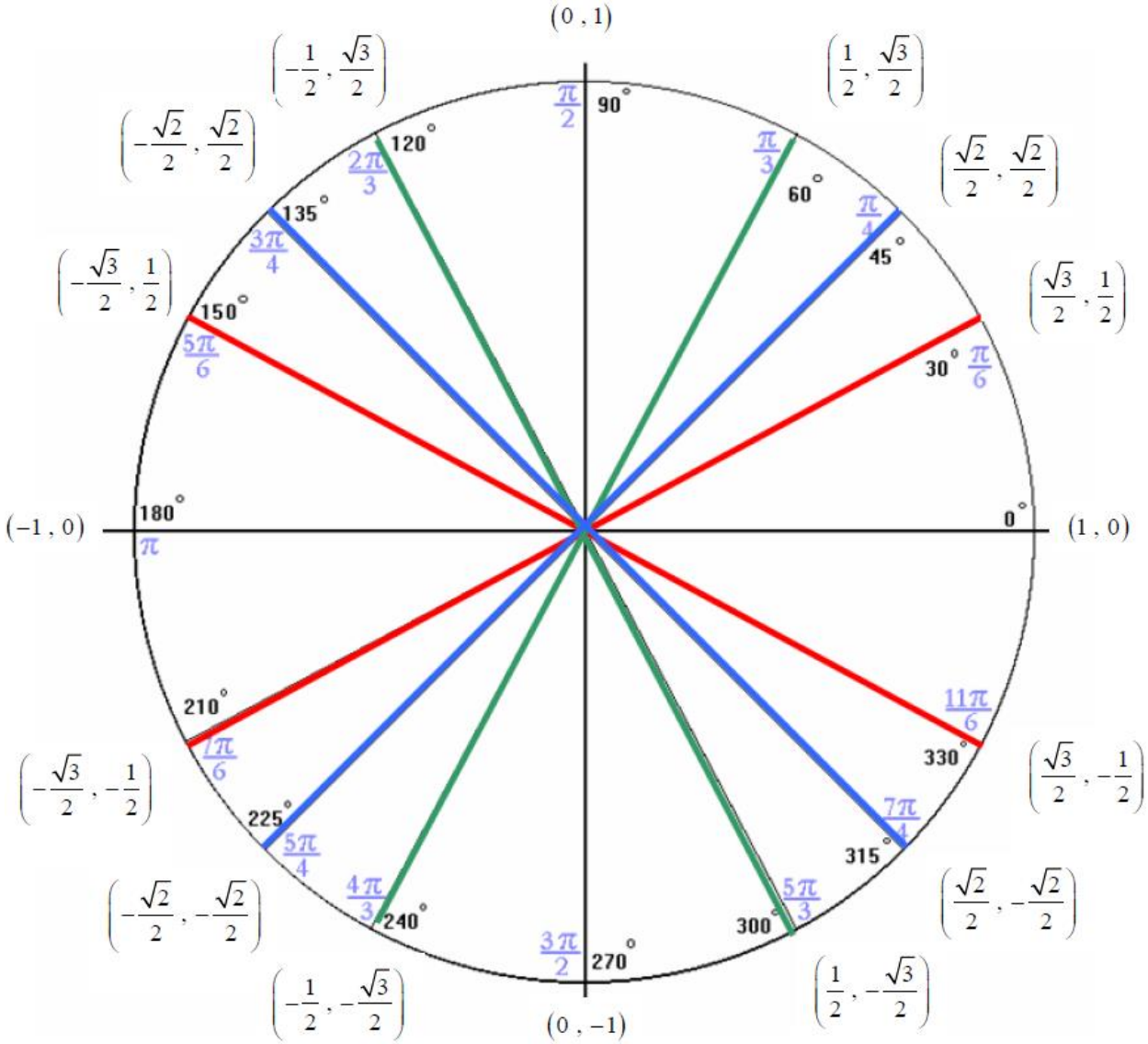
TOA $\tan\theta = \frac{OPP}{ADJ}$ $\quad \cot\theta = \frac{ADJ}{OPP}$

"All Students Take Calculus"

Students	All
Sin positive sin: + cos: - tan: -	All positive sin: + cos: + tan: +
Take	**Calculus**
Tan positive sin: - cos: - tan: +	Cos positive sin: - cos: + tan: -

The Unit Circle – the center at the origin; a radius of 1.

Coordinates of $(\cos\theta, \sin\theta)$



Patterns Formed

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\cos\theta$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{0}}{2} = 0$
$\tan\theta$	$\frac{0}{1} = 0$	$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$	$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$	$\frac{1}{0} = DNE$

Notes on patterns:

Sin θ: start at 0 under the radical and increase to 4, simplify.
Cos θ: start at 4 under the radical and decrease to 0, simplify.
Tan θ: Sin θ/ Cos θ ; simplify as appropriate.
Changing signs as needed for each quadrant.